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Nonlinear Identification Using Single Input Connected Fuzzy Inference Model

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Abstract

The single input connected fuzzy inference model (SIC model) by Hayashi et al. can decrease the number of fuzzy rules drastically in comparison with the conventional fuzzy inference models. In this paper, we first show the SIC model and its learning algorithm, and clarify the applicability of the SIC model by applying it to identification of nonlinear functions.

Keywords: Approximate reasoning; Fuzzy inference systems; Single Input Connected (SIC) fuzzy inference model; Nonlinear identification

1. Introduction

As for the "IF-THEN" rules in the conventional fuzzy inference methods [1], all the input items of the system are set to the antecedent part, and all output items are set to the consequent part. Therefore, the problem is apparent that the number of fuzzy rules becomes increasingly huge; hence, the setup and adjustment of fuzzy rules become difficult. On the other hand, a *single input rule modules connected type fuzzy inference model* (SIRMs model) by Yubazaki *et al.* [2, 3, 4, 5, 6, 7, 8, 9] which unifies the inference output from fuzzy rule modules of one input type "IF-THEN" form can reduce the number of fuzzy rules drastically. The method has been applied to nonlinear function identification, control of a first order lag system with dead time, orbital pursuit control of a non-restrained object, and stabilization control of a handstand system etc., and good results are obtained. the On the other hand, Hayashi *et al* [9, 10, 11] have also proposed a *single input connected fuzzy inference method* (SIC model) as single input type fuzzy inference method. However, since the number of rules of the SIC model is limited compared to the traditional inference models, inference results gained by the SIC method are simple in general.

In this paper, we first explain learning algorithm of the SIC model, and show the applicapility of the model by applying it to two nonlinear functions identification.

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2. Single Input Connected (SIC) Fuzzy Inference Model

In this section we review the *Single Input Connected fuzzy inference model* (SIC model) for the single input type fuzzy inference model proposed by Hayashi et al. [9, 10, 11].

The SIC model has n rule modules. Rule modules of the SIC model are given as

$$\begin{aligned} &\text{Rules-}1 : \{x_1 = A_j^1 \longrightarrow y_1 = y_j^1\}_{j=1}^{m_1} \\ &\qquad\vdots \\ &\text{Rules-}i : \{x_i = A_j^i \longrightarrow y_i = y_j^i\}_{j=1}^{m_i} \\ &\qquad\vdots \\ &\text{Rules-}n : \{x_n = A_j^n \longrightarrow y_n = y_j^n\}_{j=1}^{m_n} \end{aligned} \tag{1}$$

where Rules-i stands for the ith single input rule module, the ith input item x_i is the sole variable of the antecedent part of the Rules-i, and y_i stands for the variable of its consequent part. A_j^i means the fuzzy set of the jth rule of the Rules-i, y_j^i stands for a real value of consequent part, $i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m_i$, and m_i is the number of rules in Rules-i.

The SIC model sets up rule modules to each input item. The final inference result of the SIC model is obtained by the weighted average of the degrees of the antecedent part and consequent part of each rule module.

Degree h_j^i of the ith rule of the SIC model is given as

$$h_j^i = A_j^i(x_i^0) \tag{2}$$

The final inference result y_0 is given as follows by using degrees of antecedent part and consequent part from each rule module.

$$\begin{aligned} y_0 &= \frac{\displaystyle\sum_{j=1}^{m_1} h_j^1 y_j^1 + \cdots + \sum_{j=1}^{m_n} h_j^n y_j^n}{\displaystyle\sum_{j=1}^{m_1} h_j^1 + \cdots + \sum_{j=1}^{m_n} h_j^n} \\ &= \frac{\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m_i} h_j^i y_j^i}{\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m_i} h_j^i} \end{aligned} \tag{3}$$

3. Learning Algorithms for SIC Model

Generally speaking, the setup of membership functions and fuzzy rules is difficult. Hence, we expect to automatically optimize membership functions and fuzzy rules based on input-output data in systems. From the reason, learning algorithms for membership functions and fuzzy rules are proposed in [14, 15, 16, 17, 18, 19]. In this section, we review a learning algorithm of the SIC inference model from the steepest descent method [20]. As for the learning method, the parameters are learned for the membership functions of the antecedent parts and consequent parts.

When the training input–output data $(x_{r_1}, x_{r_2}, \ldots, x_{r_m};\ y^{T_r})$ are given for a fuzzy system model, it is usual to use the following objective function E_r for evaluating an error between y^{T_r} and y^{0_r}, which can be regarded as an optimum problem:

$$E_r = \frac{1}{2}(y^{T_r} - y^{0_r})^2 \tag{4}$$

where y^{T_r} is the desired output value, and y^{0_r} the corresponding fuzzy inference result.

The triangular-type and Gaussian-type fuzzy sets are used as two kinds of fuzzy sets. The parameters of center a^i_j and width b^i_j of the fuzzy sets, and consequent part $f^i_j(x_i)$ are obtained by the steepest descent method as follows [20].

(I) Case of triangular-type fuzzy sets:

We consider the following triangular-type fuzzy set $A^i_j(x_i)$.

$$A^i_j(x_i) = \begin{cases} 1 - |x_i - a^i_j|/b^i_j; & a^i_j - b^i_j \le x_i \le a^i_j + b^i_j \\ 0; & \text{otherwise} \end{cases} \tag{5}$$

where a^i_j and b^i_j ($i = 1, 2, \ldots, n$; $j = 12, \ldots, m_i$) stand for the center and width, respectively. From (5), the learning algorithms at $t + 1$ step of each parameter are obtained as follows.

$$a^i_j(t+1) = a^i_j(t) + \alpha \cdot (y^T - y^0(t)) \cdot \frac{y^i_j(t) - y^0(t)}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h^i_k(t)} \cdot \frac{\mathrm{sgn}(x_i - a^i_j(t))}{b^i_j(t)} \tag{6}$$

$$b^i_j(t+1) = b^i_j(t) + \beta \cdot (y^T - y^0(t)) \cdot \frac{y^i_j(t) - y^0(t)}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h^i_k(t)} \cdot \frac{|x_i - a^i_j(t)|}{(b^i_j(t))^2} \tag{7}$$

$$y^i_j(t+1) = c^i_j(t) + \gamma \cdot (y^T - y^0(t)) \cdot \frac{h^i_j(t)}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h^i_k(t)} \tag{8}$$

where α, β, γ and δ are the learning rates in the learning process, and t means the learning iteration number.

(II) Case of Gaussian-type fuzzy sets:

We consider the following Gaussian-type fuzzy set $A^i_j(x_i)$.

$$A^i_j(x_i) = \exp\left(-\frac{(x_i - a^i_j)^2}{b^i_j}\right) \tag{9}$$

where a^i_j and b^i_j ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m_i$) stand for the center and width, respectively. From (9), the learning algorithms of each parameter are obtained as follows.

$$a^i_j(t+1) = a^i_j(t) + \alpha \cdot (y^T - y^0(t)) \cdot (y^i_j(t) - y^0(t)) \cdot \frac{h^i_j(t)}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h^i_k(t)} \cdot \frac{2(x_i - a^i_j(t))}{b^i_j(t)} \cdot h^i_j(t) \cdot \frac{2(x_i - a^i_j(t))}{b^i_j(t)} \tag{10}$$

$$b^i_j(t+1) = b^i_j(t) + \beta \cdot (y^T - y^0(t)) \cdot (y^i_j(t) - y^0(t)) \cdot \frac{h^i_j(t)}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h^i_k(t)} \cdot \left(\frac{x_i - a^i_j(t)}{b^i_j}\right)^2 \tag{11}$$

$$y^i_j(t+1) = c^i_j(t) + \gamma \cdot (y^T - y^0(t)) \cdot \frac{h^i_j(t)}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h^i_k(t)} \tag{12}$$

where $\alpha, \beta, \gamma, \delta$ and t have the same meanings as the case of the triangular-type fuzzy sets.

4. Identification of Nonlinear Functions by SIC Model

In this section, we apply the SIC model and the above learning algorithms to the following two nonlinear functions with two input variables and one output variable.

$$\text{Function1.} \qquad y = \frac{(2x_1 + 4x_2^2 + 0.1)^2}{37.21} \tag{13}$$

$$\text{Function2.} \quad y = \frac{(2\sin(\pi x_1) + \cos(\pi x_2) + 3)}{6} \tag{14}$$

where $x_1,\ x_2 \in [-1, 1]$ are input variavles, and $y \in [0, 1]$ is a normalized output variable.

In identifying nonlinear functions, there are five membership functions for the inputs x_1 and x_2, where the centers of the membership functions $A_1^i, A_2^i, \ldots, A_5^i$ for $i = 1,\ 2$ are -1, -0.5, 0, 0.5, 1, and each width of membership functions is 0.5. Moreover, all of consequent parts are set to be 0.

Here, we obtain the error of evaluation regarding desired model and inference model where the error of evaluation is mean square error for checking data.

In our case, 2601 checking data $(x_1,\ x_2)$ are employed from $(-1, -1)$ to $(1, 1)$, and 49 training data are used from 2601 checking data in a random order. The learning rates are $\alpha = 0.0001, \beta = 0.00001$ and $\gamma = 0.01$.

In the following, we identify Functions 1 and 2 by using the SIC model (SIC, for short in the tables) in the case of the triangular-type and Gaussian-type membership functions, respectively.

For nonlinear functions 1 and 2, learninig iterations are executed 1000 times, and 10 simulations are run. Table 1 shows the error of evaluation using the checking data in the case of the triangular-type and Gaussian-type membership functions for identifying Functions 1, respectively. Table 2shows the error of evaluation using the checking data in the case of the triangular-type and Gaussian-type membership functions for identifying Functions 2, respectively.

Table 1. Error of evaluation for Function 1 of (13)

Case	Triangular-type	Gaussian-type
1	0.010158	0.010233
2	0.009568	0.010794
3	0.01056	0.010726
4	0.010695	0.009058
5	0.010659	0.012592
6	0.011395	0.010525
7	0.009568	0.011587
8	0.010942	0.012396
9	0.009304	0.009587
10	0.011627	0.010771
Average	0.010448	0.010827

Table 2. Error of evaluation for Function 2 of (14)

Case	Triangular-type	Gaussian-type
1	0.001737	0.010559
2	0.001611	0.012281
3	0.002055	0.01131
4	0.001593	0.010822
5	0.002331	0.01259
6	0.002058	0.01066
7	0.001997	0.011654
8	0.0018	0.01137
9	0.001649	0.011271
10	0.002124	0.010386
Average	0.001896	0.01129

The SIC model do not necessarily obtain good results comapred with the neuro-fuzzy method based on the simplified fuzzy inference method for the Function 1 as a multiplicative function, as shown in Table 1. On the other hand, all methods give good results regarding Function 2 as an additive function, as shown in Table 2.

From these results, we have clarified that the SIC model can obtain good results for additive functions even if a few rules are used.

Moreover, the SIC model uses 10 (= 5×2) rules though the conventioanl fuzzy inference models uses 25 (= 5^2) rules.

Therefore, we have shown the applicability of the SIC model for nonlinear functions.

5. Conclusions

In this paper, we have shown the applicability of the SIC model by applying to two nonlinear functons identification. Although the number of rules of the SIC model is few, it has obtained good results. Further studies are required to optimize the number of fuzzy sets and the parameters of the SIC model.

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