

Distributed Coordination of Networked Fractional-Order Systems

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Abstract—This paper studies the distributed coordination of networked fractional-order systems over a directed interaction graph. A general fractional-order coordination model is introduced by summarizing three different cases: 1) fractional-order agent dynamics with integer-order coordination algorithms; 2) fractional-order agent dynamics with fractional-order coordination algorithms; and 3) integer-order agent dynamics with fractional-order coordination algorithms. We show sufficient conditions on the interaction graph and the fractional order such that coordination can be achieved using the general model. The coordination equilibrium is also explicitly given. In addition, we characterize the relationship between the number of agents and the fractional order to ensure coordination. Furthermore, we compare the convergence speed of coordination for fractional-order systems with that for integer-order systems. It is shown that the convergence speed of the fractional-order coordination algorithms can be improved by varying the fractional orders with time. Finally, simulation results are presented as a proof of concept.

Index Terms—Consensus, coordination, directed graph, fractional-order systems, multiagent systems.

I. INTRODUCTION

CCOORDINATION of multiagent systems has numerous applications. Examples include rendezvous, flocking, formation stabilization, and sensor networks. Distributed coordination aims at achieving collective group behavior through local interaction. Consensus plays an important role in distributed coordination. Consensus has an old history [1]–[3]. In the literature, consensus denotes the agreement of a group faced with decision-making situations. As for a group behavior, sharing information with each other or consulting more than one expert, as stated in [1], makes the decision makers more confident.

A. Related Work

In the control community, consensus algorithms have extensively been studied. The objective is to reach an agreement on

information states, including positions, velocities, and attitudes, via local interaction. Consensus algorithms have applications in rendezvous, formation control, and sensor networks. For systems with single-integrator dynamics, consensus algorithms have been studied in both continuous-time and discrete-time settings (see [4] and references therein). Because many vehicles are modeled by double-integrator dynamics, consensus algorithms for double-integrator dynamics are studied in [5]–[10], to name a few.

Fractional calculus can be dated back to the seventeenth century [11]. Fractional calculus studies fractional derivatives, fractional integrals, and their properties. Different from the integer orders of derivatives and integrals in conventional calculus, the orders of derivatives and integrals in fractional calculus are real numbers. The foundations of fractional calculus were laid on [12]–[14]. With the development of fractional calculus, its applications were also studied by researchers from different disciplines [15], [16]. Examples include the study of the formation of particulate aggregates [17] and the study of motion of objects in viscoelastic materials [18]–[20]. In particular, fractional calculus was also introduced into the engineering community to design a CRONE controller [21] and synthesize control systems [22], to name a few.

In addition, fractional dynamics were also presented and studied from different perspectives. The authors in [23] modeled the dynamics of self-similar protein in a fractional-order sense because the relaxation processes and the reaction kinetics of proteins deviated from the exponential behavior. In [24], the fractional-order dynamics of international commodity prices were demonstrated from the commodity price series. In [25] and [26], the authors studied the fractional-order proportional-integral differential (PID) controllers, which show better performance when used for the control of fractional-order systems than the classical PID controllers. The authors in [27] demonstrated that fractional equations have become a complementary tool in the description of anomalous transport processes in complex systems.

B. Motivation of This Paper

Many phenomena in nature cannot be explained in the framework of integer-order dynamics, for example, the synchronized motion of agents in fractional circumstances, such as macromolecule fluids and porous media. Under these circumstances, the stress-strain relationship demonstrates non-integer-order (i.e., fractional-order) dynamics rather than integer-order dynamics, as shown in [18]–[20]. In addition, many other phenomena can naturally be explained by the coordinated behavior

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of agents with fractional-order dynamics. Examples include chemotaxis behavior and food seeking of microbes and collective motion of bacteria in lubrications perspired by themselves [28], [29]. Similarly, engineered systems often demonstrate fractional-order dynamics either because the environments in which they are operated are complex or because the system dynamics can more accurately be modeled by fractional-order differential equations than integer-order differential equations. Examples include underwater vehicles operating in lentic lakes composed of microbes and viscoelastic materials, flying vehicles operating in an environment where the influence of particles in air cannot be ignored (e.g., high-speed flight in duststorm, rain, or snow), and ground vehicles moving on top of carpet, sand, muddy road, or grass. In addition, friction in many real-world applications takes in the form of a fractional-order model instead of an integer-order model.

Motivated by the broad application of coordination algorithms in multivehicle systems and the fact that many practical vehicles demonstrate fractional dynamics, we study coordination algorithms for networked fractional-order systems in this paper. To the best of our knowledge, this paper is the first paper that studies the distributed coordination of networked fractional-order systems.

C. Contribution of This Paper

The contribution of this paper is as follows. First, the coordination for integer-order dynamics is extended to fractional-order dynamics. The existing coordination algorithms for integer-order dynamics can be considered a special case of fractional-order coordination algorithms. Second, we study the conditions on the interaction graph and the fractional order such that coordination can be achieved for networked fractional-order systems. We also characterize the relationship between the number of agents and the fractional order to ensure coordination. Third, an alternative to improve the convergence speed is presented by applying a varying-order coordination strategy.

D. Organization of This Paper

The remainder of this paper is organized as follows. In Section II, graph theory notions and the Caputo fractional operator are introduced. The main results on coordination for fractional-order systems are presented in Section III. Then, the comparison between coordination for integer-order systems and fractional-order systems is given in Section IV. Finally, simulation results and a short conclusion are given in Sections V and VI, respectively. A preliminary version of this paper has appeared in [30].

II. PRELIMINARY

In this section, we introduce graph theory notions and the Caputo fractional operator, which serve as a basis for the following several sections.

A. Graph Theory Notions

For a system with n agents, the interaction graph for all agents can be modeled by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{W})$, where

$\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ and $\mathcal{W} \subseteq \mathcal{V}^2$ represent the agent set and the edge set of the graph, respectively. Each edge can be denoted as (v_i, v_j) , which means agent j can access the state information of agent i . Accordingly, agent i is a neighbor of agent j . All neighbors of agent i are denoted by N_i . A directed path is a sequence of edges in a directed graph with the form $(v_1, v_2), (v_2, v_3), \dots$, where $v_i \in \mathcal{V}$. A directed graph has a directed spanning tree if there exists at least one agent that has a directed path to all the other agents.

The interaction graph can be represented by two types of matrices: 1) the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ with $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{W}$ and $a_{ij} = 0$ otherwise, and 2) the (nonsymmetric) Laplacian matrix $L = [\ell_{ij}] \in \mathbb{R}^{n \times n}$ with $\ell_{ii} = \sum_{j \in N_i} a_{ij}$ and $\ell_{ij} = -a_{ij}$, $i \neq j$. It is straightforward to verify that L has at least one zero eigenvalue with a corresponding eigenvector $\mathbf{1}$, where $\mathbf{1}$ is an all-one column vector with a compatible size.

Lemma 1 [31], [32]: For a fixed interaction graph, L has a simple zero eigenvalue with an associated eigenvector $\mathbf{1}$, and all the other eigenvalues have positive real parts if and only if the interaction graph has a directed spanning tree.

B. Caputo Fractional Operator

There are mainly two widely used fractional operators: Caputo and Riemann–Liouville (R–L) fractional operators [16]. In physical systems, the Caputo fractional operator is more practical than the R–L fractional operator because the R–L fractional operator has initial value problems. Therefore, we will use the Caputo fractional operator in this paper to model the system dynamics and analyze the stability of the proposed fractional-order algorithms. In the following section, we will review the Caputo fractional operator. Generally, the Caputo fractional operator includes Caputo integral and Caputo derivative. The Caputo derivative is defined based on the following Caputo integral:

$${}_a^C D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau$$

where ${}_a^C D_t^{-\alpha}$ denotes the Caputo integral with order $\alpha \in (0, 1]$, $\Gamma(\cdot)$ is the Gamma function, and a is an arbitrary real number. For any real number p , the Caputo derivative is defined as

$${}_a^C D_t^p f(t) = {}_a^C D_t^{-\alpha} \left[\frac{d^{[p]+1}}{dt^{[p]+1}} f(t) \right] \quad (1)$$

where $\alpha = [p] + 1 - p \in (0, 1]$, and $[p]$ is the integer part of p . If p is an integer, then $\alpha = 1$, and (1) is equivalent to the integer-order derivative. Because only the Caputo fractional operator is used in this paper, a simple notation $f^{(\alpha)}(t)$ is used to replace ${}_a^C D_t^\alpha f(t)$.

In the following, we will introduce the Laplace transform of the Caputo derivative and the Mittag–Leffler function [33], which will be used to, respectively, analyze the algorithms proposed in Section III and make a comparison between coordination for fractional-order systems and integer-order systems in Section IV. We first introduce the Laplace transform of the

Caputo derivative. Let $\mathcal{L}\{\cdot\}$ denote the Laplace transform of a function. It follows from the formal definition of the Laplace transform $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$ that

$$\mathcal{L}\{f^{(\alpha)}(t)\} = \begin{cases} s^\alpha F(s) + s^{\alpha-1} f(0^-), & \alpha \in (0, 1] \\ s^\alpha F(s) + s^{\alpha-1} f(0^-) \\ \quad + s^{\alpha-2} \dot{f}(0^-), & \alpha \in (1, 2] \end{cases}$$

where $f(0^-) = \lim_{\epsilon \rightarrow 0^-} f(\epsilon)$, and $\dot{f}(0^-) = \lim_{\epsilon \rightarrow 0^-} \dot{f}(\epsilon)$. We then introduce the Mittag-Leffler function, which is a function frequently used in the solutions of fractional-order systems, as shown in Section IV. For $\alpha, \beta \in \mathbb{C}$, the Mittag-Leffler function in two parameters is defined as

$$E_{\alpha, \beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha + \beta)}. \quad (2)$$

When $\beta = 1$, and $\alpha > 0$, (2) can be written in a special case as

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha + 1)}. \quad (3)$$

III. STABILITY ANALYSIS OF COORDINATION ALGORITHMS FOR FRACTIONAL-ORDER SYSTEMS

In this section, we derive the conditions on the interaction graph and the fractional order such that the coordination for fractional-order systems is achieved over a directed fixed interaction graph. To study this problem, we introduce a general model of coordination for fractional-order systems by summarizing the following three different cases.

Case (i): Fractional-order agent dynamics with an integer-order coordination algorithm: Assume that the agent dynamics are

$$x_i^{(\alpha)}(t) = u_i(t) \quad (4)$$

where $x_i(t)$ and $u_i(t)$ represent, respectively, the state and the control input for the i th agent, and $x_i^{(\alpha)}(t)$ is the α th derivative of $x_i(t)$ with $\alpha \in \mathbb{R}^+$.¹ An integer-order coordination algorithm is given by

$$u_i(t) = \sum_{j \in N_i} a_{ij} \{ [x_j(t) - \delta_j] - [x_i(t) - \delta_i] \} \quad (5)$$

where a_{ij} is the (i, j) th entry of the adjacency matrix A , N_i denotes the neighbor set of agent i , and δ_i is a constant.

Case (ii): Integer-order agent dynamics with a fractional-order coordination algorithm: Assume that the agent dynamics are given by $\dot{x}_i(t) = u_i(t)$, where

$x_i(t)$ and $u_i(t)$ are defined as in (4). A fractional-order coordination algorithm is given by

$$u_i(t) = \sum_{j \in N_i} a_{ij} \{ [x_j(t) - \delta_j]^{(\beta)} - [x_i(t) - \delta_i]^{(\beta)} \} \quad (6)$$

where $\beta \in \mathbb{R}^+$, and a_{ij} , N_i , and δ_i are defined as in (5).

Case (iii): Fractional-order agent dynamics with a fractional-order coordination algorithm: Assume that the agent dynamics are given by (4). A fractional-order coordination algorithm is given by (6).

Define $\delta_{ij} \triangleq \delta_i - \delta_j$. The objective of the algorithm in each case is to guarantee coordination, i.e., $x_i(t) - x_j(t) \rightarrow \delta_{ij}$ as $t \rightarrow \infty$ for any initial $x_i(0)$ and $x_j(0)$. Note that integer-order dynamics [i.e., α is an integer in (4)] is a special case of fractional-order dynamics. The existing consensus algorithm for single-integrator dynamics (e.g., [32], [34], [35]) corresponds to a special case of Case (i) when $\alpha = 1$ in (4) and $\delta_{ij} = 0$ in (5).

When applying the Caputo derivative to (4) and (6), it follows that Cases (ii) and (iii) can be written as Case (i) by applying the fractional operator ${}_a^C D_t^{-\beta}$ on both sides of the corresponding system. Therefore, the model in Case (i) can be considered a general model. In the following, we focus on Case (i). For an n -agent system, using (5), (4) can be written in matrix form as

$$\tilde{X}^{(\alpha)}(t) = -L\tilde{X}(t) \quad (7)$$

where $\tilde{X}(t) = [\tilde{x}_1(t), \tilde{x}_2(t), \dots, \tilde{x}_n(t)]^T \in \mathbb{R}^n$ with $\tilde{x}_i(t) = x_i(t) - \delta_i$, and L is the (nonsymmetric) Laplacian matrix. Although the dynamics for a given system are fixed, α in the general model (7) can be changed by choosing coordination algorithms with different fractional orders.

Note that L can be written in Jordan canonical form as

$$L = P \underbrace{\begin{bmatrix} \Lambda_1 & 0 & \cdots & 0 \\ 0 & \Lambda_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \Lambda_k \end{bmatrix}}_{\Lambda} P^{-1}$$

where Λ_m , $m = 1, 2, \dots, k$, are standard Jordan blocks. Without loss of generality, let the initial time $a = 0$. By defining $Y(t) \triangleq P^{-1}\tilde{X}(t)$, (7) can be written as

$$Y^{(\alpha)}(t) = -\Lambda Y(t). \quad (8)$$

Suppose that the diagonal entry of Λ_i is λ_i (i.e., an eigenvalue of L). Noting that the standard Jordan block $\Lambda_i =$

$\begin{bmatrix} \lambda_i & 1 & \cdots & 0 \\ 0 & \lambda_i & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda_i \end{bmatrix}$, it follows that (8) can be decoupled into n one-dimensional equations represented by either

$$y_i^{(\alpha)}(t) = -\lambda_i y_i(t) \quad (9)$$

¹For a given system, α is fixed.

for the equation corresponding to Λ_i , which has dimension equal to 1, or the last equation corresponding to Λ_i , which has dimension larger than 1, or

$$y_i^{(\alpha)}(t) = -\lambda_i y_i(t) - y_{i+1}(t) \quad (10)$$

otherwise, where $y_i(t)$ is the i th component of $Y(t)$.

Before deriving the main result, we need the following two lemmas.

Lemma 2: When $\text{Re}(\lambda_i) \geq 0$, where $\text{Re}(\cdot)$ denotes the real part of a complex number, the solution of (9) has the following properties.

- 1) When $\alpha \in (0, (2\theta_i/\pi))$, and $\text{Re}(\lambda_i) > 0$, $\lim_{t \rightarrow \infty} y_i(t) \rightarrow 0$ as $t \rightarrow \infty$, where $\theta_i = \pi - \arg\{\lambda_i\}$, with $\arg\{\lambda_i\}$ denoting the phase of λ_i .²
- 2) When $\alpha \in (0, 1]$, and $\lambda_i = 0$, $y_i(t) \equiv y_i(0) \forall t$.
- 3) When $\alpha \in (1, 2)$, and $\lambda_i = 0$, $y_i(t) = y_i(0) + \dot{y}_i(0)t$.
- 4) When $\alpha \in (2, \infty)$, the system is not stable.

Proof: (Proof of Property 1) By taking the Laplace transform of (9), it can be computed from the Laplace transform of $\mathcal{L}\{f^{(\alpha)}(t)\}$ in Section II-B that

$$\mathcal{L}\{y_i(t)\} = \frac{y_i(0^-)s^{\alpha-1}}{s^\alpha + \lambda_i}, \quad \alpha \in (0, 1] \quad (11)$$

$$\mathcal{L}\{y_i(t)\} = \frac{y_i(0^-)s^{\alpha-1} + \dot{y}_i(0^-)s^{\alpha-2}}{s^\alpha + \lambda_i}, \quad \alpha \in (1, 2). \quad (12)$$

From (11) and (12), it can be seen that the denominator of $\mathcal{L}\{y_i(t)\}$ is $s^\alpha + \lambda_i$ when $\alpha \in (0, 2)$. To ensure that all the poles of $\mathcal{L}\{y_i(t)\}$ are in the open left half plane (LHP), it follows from the discussion in [36] that $\alpha \in (0, (2(\pi - \arg\{\lambda_i\})/\pi))$, that is, $\alpha \in (0, (2\theta_i/\pi))$, where $(2\theta_i/\pi) > 1$ because $\text{Re}(\lambda_i) > 0$, i.e., $\arg\{\lambda_i\} \in (-(\pi/2), (\pi/2))$. In particular, when $\lambda_i \in \mathbb{R}^+$, $\alpha \in (0, 2)$ because $\arg\{\lambda_i\} = 0$.

(Proof of Properties 2 and 3) The proofs of Properties 2 and 3 follow from [16].

(Proof of Property 4) See [33]. ■

Lemma 3: Assume that continuous function $y_{i+1}(t)$ satisfies $\lim_{t \rightarrow \infty} y_{i+1}(t) = 0$. When $\text{Re}(\lambda_i) > 0$ [i.e., $\arg\{\lambda_i\} \in (-(\pi/2), (\pi/2))$], and $\alpha \in (0, (2\theta_i/\pi))$, where $\theta_i = \pi - \arg\{\lambda_i\}$, the solution of (10) satisfies $\lim_{t \rightarrow \infty} y_i(t) = 0$.

Proof: When $\alpha \in (0, 1]$, by taking the Laplace transform of (10), it can be computed from the Laplace transform of $\mathcal{L}\{f^{(\alpha)}(t)\}$ that

$$\mathcal{L}\{y_i(t)\} = \frac{s^{\alpha-1}y_i(0^-) - \mathcal{L}\{y_{i+1}(t)\}}{s^\alpha + \lambda_i}. \quad (13)$$

It follows from the proof of Property 1 in Lemma 2 that the poles of (13) are in the open LHP when $\alpha \in (0, 1]$. By applying the final value theorem of the Laplace transform, we have

$$\begin{aligned} \lim_{t \rightarrow \infty} y_i(t) &= \lim_{s \rightarrow 0} s \mathcal{L}\{y_i(t)\} \\ &= \lim_{s \rightarrow 0} \frac{s^\alpha y_i(0^-) - s \mathcal{L}\{y_{i+1}(t)\}}{s^\alpha + \lambda_i} = 0 \end{aligned}$$

²We follow the convention that $\arg\{x\} \in (-\pi, \pi]$ for $x \in \mathbb{C}$.

where we have used the fact $s\mathcal{L}\{y_{i+1}(t)\} = 0$ to derive the last equality because $\lim_{t \rightarrow \infty} y_{i+1}(t) = 0$.

When $\alpha \in (1, (2\theta_i/\pi))$, it follows from the proof of Property 1 in Lemma 2 that the poles of (13) are also in the open LHP. By taking the Laplace transform of (10), it can be computed from the Laplace transform of $\mathcal{L}\{f^{(\alpha)}(t)\}$ that

$$\mathcal{L}\{y_i(t)\} = \frac{s^{\alpha-1}y_i(0^-) + s^{\alpha-2}\dot{y}_i(0^-) - \mathcal{L}\{y_{i+1}(t)\}}{s^\alpha + \lambda_i}. \quad (14)$$

Following a similar discussion for $\alpha \in (0, 1]$ gives $\lim_{t \rightarrow \infty} y_i(t) = 0$.

Combining the foregoing arguments proves the lemma. ■

Based on Lemmas 2 and 3, we next study the conditions on the fractional order α and the interaction graph such that coordination can be achieved.

Theorem 1: Let λ_i be the i th eigenvalue of L , and $\theta = \min_{\lambda_i \neq 0, i=1,2,\dots,n} \theta_i$, where $\theta_i = \pi - \arg\{\lambda_i\}$. For the fractional-order system [see (7)], coordination is achieved if the fixed interaction graph has a directed spanning tree and $\alpha \in (0, 2\theta/\pi)$. When $\alpha \in (0, 1]$, the solution of (7) satisfies $\tilde{x}_i(t) \rightarrow \tilde{x}_j(t) \rightarrow \mathbf{p}^T \tilde{X}(0)$, i.e., $x_i(t) - x_j(t) \rightarrow \delta_{ij}$ as $t \rightarrow \infty$, where \mathbf{p} is the left eigenvector of L associated with the zero eigenvalue satisfying $\mathbf{p}^T \mathbf{1} = 1$. When $\alpha \in (1, 2\theta/\pi)$, the solution of (7) satisfies $\tilde{x}_i(t) \rightarrow \tilde{x}_j(t) \rightarrow \mathbf{p}^T \tilde{X}(0) + \mathbf{p}^T \dot{\tilde{X}}(0)t$ and $\dot{\tilde{x}}_i(t) \rightarrow \dot{\tilde{x}}_j(t) \rightarrow \mathbf{p}^T \dot{\tilde{X}}(0)$, i.e., $x_i(t) - x_j(t) \rightarrow \delta_{ij}$, as $t \rightarrow \infty$.

Proof: Noting that the interaction graph has a directed spanning tree, it follows from Lemma 1 that L has a simple zero eigenvalue, and all other eigenvalues have positive real parts. Without loss of generality, let $\lambda_1 = 0$, and $\text{Re}(\lambda_i) > 0$, $i \neq 1$. When $\alpha \in (0, 1]$, because $\lambda_1 = 0$ is a simple zero eigenvalue, λ_1 satisfies (9). It follows from Property 2 in Lemma 2 that $y_1(t) \equiv y_1(0)$. When λ_i , $i \neq 1$, satisfies (9), it follows from Property 1 in Lemma 2 that $\lim_{t \rightarrow \infty} y_i(t) = 0$, $i \neq 1$. When λ_i , $i \neq 1$, satisfies (10), it follows from Lemma 3 that $\lim_{t \rightarrow \infty} y_i(t) = 0$, $i \neq 1$, as well because $y_{i+1}(t)$ also satisfies either (9) or (10), which implies $\lim_{t \rightarrow \infty} y_{i+1}(t) = 0$. Combining the foregoing arguments gives $\lim_{t \rightarrow \infty} Y(t) = [y_1(0), 0, \dots, 0]^T$, $i \neq 1$, which implies $\lim_{t \rightarrow \infty} \tilde{X}(t) = \lim_{t \rightarrow \infty} PY(t) = PSY(0) = PSP^{-1}\tilde{X}(0)$, where $S = [s_{ij}] \in \mathbb{R}^{n \times n}$ has only one nonzero entry $s_{11} = 1$. Note that the first column of P can be chosen as $\mathbf{1}$, whereas the first row of P^{-1} can be chosen as \mathbf{p} by noting that $\mathbf{1}$ and \mathbf{p} are, respectively, a right and left eigenvector of L associated with $\lambda_1 = 0$ and $\mathbf{p}^T \mathbf{1} = 1$. Therefore, $\lim_{t \rightarrow \infty} \tilde{X}(t) = PSP^{-1}\tilde{X}(0) = \mathbf{1}\mathbf{p}^T \tilde{X}(0)$, that is, $\lim_{t \rightarrow \infty} \tilde{x}_i(t) = \mathbf{p}^T \tilde{X}(0)$. This implies that $x_i(t) - x_j(t) \rightarrow \delta_{ij}$ as $t \rightarrow \infty$.

When $\alpha \in (1, (2\theta/\pi))$, similar to the previous discussion for $\alpha \in (0, 1]$, λ_1 satisfies (9). It follows from Property 3 in Lemma 2 that $y_1(t) = y_1(0) + \dot{y}_1(0)t$. Because $\text{Re}(\lambda_i) > 0$, $i \neq 1$, similar to the previous discussion for $\alpha \in (0, 1]$, it follows from Property 1 in Lemmas 2 and 3 that $\lim_{t \rightarrow \infty} y_i(t) = 0$, $i \neq 1$. Therefore, it follows that $\lim_{t \rightarrow \infty} Y(t) = [y_1(0) + \dot{y}_1(0)t, 0, \dots, 0]^T$, which implies that $\lim_{t \rightarrow \infty} \dot{Y}(t) = [\dot{y}_1(0), 0, \dots, 0]^T$. Similar to

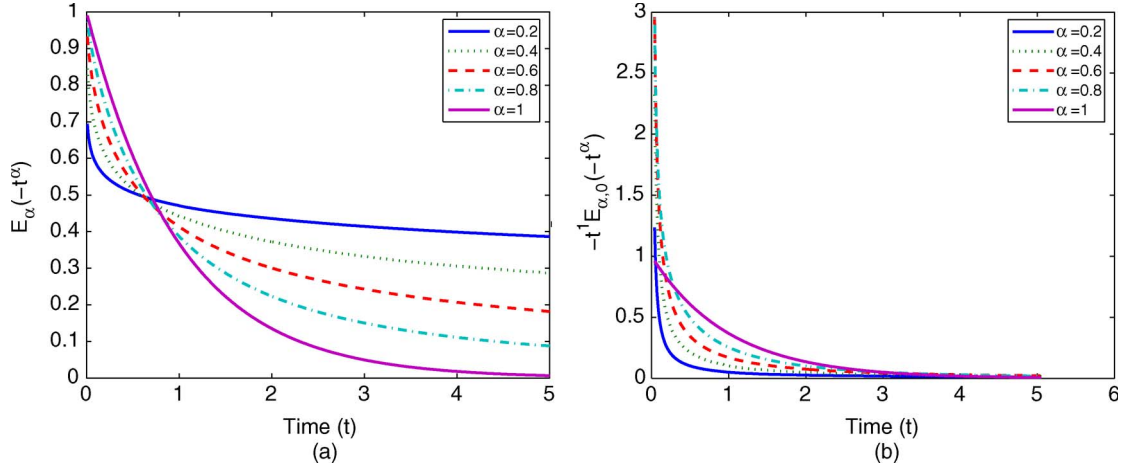


Fig. 1. Mittag-Leffler functions and the derivatives. (a) Mittag-Leffler functions with different orders. (b) Derivatives of Mittag-Leffler functions with different orders.

the proof for $\alpha \in (0, 1]$, it directly follows that $\lim_{t \rightarrow \infty} \tilde{x}_i(t) = \mathbf{p}^T \tilde{X}(0) + \mathbf{p}^T \dot{\tilde{X}}(0)t$, and $\lim_{t \rightarrow \infty} \dot{\tilde{x}}_i(t) = \mathbf{p}^T \dot{\tilde{X}}(0)$. This implies that $x_i(t) - x_j(t) \rightarrow \delta_{ij}$ as $t \rightarrow \infty$.

Combining the previous arguments for $\alpha \in (0, 1]$ and $\alpha \in (1, 2\theta/\pi)$ proves the theorem. ■

As a special case, when the fixed interaction graph is undirected, we can obtain the following result.

Corollary 1: Assume that the fixed interaction graph is undirected. For the fractional-order system [see (7)], coordination is achieved if the interaction graph is connected and $\alpha \in (0, 2)$. The coordination equilibria when $\alpha \in (0, 1]$ and $\alpha \in (1, (2\theta/\pi))$ are the same as those in Theorem 1.

Proof: When the undirected interaction graph is connected, it follows that there is a simple zero eigenvalue, and all the other eigenvalues are positive, which implies that $\theta = \pi$. The statements then follow from the proof in Theorem 1. ■

From Theorem 1, it can be seen that the range of the fractional order α is determined by θ . Note that θ is closely related to the eigenvalues of L , which are also related to the number of agents. In the following, we characterize the relationship between α and the number of agents to ensure coordination.

Theorem 2: Assume that there are n agents with $n \geq 2$. For the fractional-order system [see (7)], coordination can be achieved if the fixed interaction graph has a directed spanning tree and $\alpha \in (0, 1 + (2/n))$.

Proof: Letting λ_i be the i th eigenvalue of L , it follows from [37] that $\arg\{\lambda_i\} \in [-(\pi/2) + (\pi/n), (\pi/2) - (\pi/n)]$ for all $\lambda_i \neq 0$, which implies $(2\theta/\pi) \geq 1 + (2/n)$. Therefore, the statement apparently holds for Theorem 1. ■

Remark 1: From Theorem 1, it can be seen that the final coordination equilibrium of (7) for $\alpha \in (0, 1]$ is the same as that of

$$\dot{\tilde{X}}(t) = -L\tilde{X}(t) \quad (15)$$

under the same L .

Remark 2: From Theorem 2, when there exist more agents in a team (i.e., n becomes larger), α has to be chosen smaller to ensure coordination. As $n \rightarrow \infty$, $2\theta/\pi \rightarrow 1$, i.e., $\alpha \in (0, 1]$,

which implies that the coordination property for systems with single-integrator dynamics does not depend on n .

IV. COMPARISON BETWEEN COORDINATION FOR FRACTIONAL-ORDER SYSTEMS AND INTEGER-ORDER SYSTEMS

In this section, we compare the coordination for fractional-order systems with that for integer-order systems. Based on the comparison, we propose a varying-order fractional-order coordination strategy to achieve higher convergence speed. Before moving on, we first derive the solutions of (9) and (10).

For $\alpha \in (0, 1]$, the Laplace transform of (9) is (11). Taking the inverse Laplace transform of (11) gives

$$y_i(t) = y_i(0^-)E_\alpha(-\lambda_i t^\alpha)$$

where $E_\alpha(\cdot)$ is the Mittag-Leffler function defined in (3). Similarly, for $\alpha \in (1, 2)$, the Laplace transform of (9) is (12). Taking the inverse Laplace transform of (12) gives

$$y_i(t) = y_i(0^-)E_\alpha(-\lambda_i t^\alpha) + \dot{y}_i(0^-)tE_{\alpha,2}(-\lambda_i t^\alpha)$$

where $E_{\alpha,2}(\cdot)$ is the Mittag-Leffler function defined in (2).

For $\alpha \in (0, 1]$, the Laplace transform of (10) is (13). Taking the inverse Laplace transform of (13) gives

$$y_i(t) = y_i(0^-)E_\alpha(-\lambda_i t^\alpha) - y_{i+1}(t) * [t^{\alpha-1}E_{\alpha,\alpha}(-\lambda_i t^\alpha)]$$

where $*$ denotes the convolution operation. Similarly, for $\alpha \in (1, 2)$, the Laplace transform of (10) is (14). Taking the inverse Laplace transform of (14) gives

$$y_i(t) = y_i(0^-)E_\alpha(-\lambda_i t^\alpha) + \dot{y}_i(0^-)tE_{\alpha,2}(-\lambda_i t^\alpha) - y_{i+1}(t) * [t^{\alpha-1}E_{\alpha,\alpha}(-\lambda_i t^\alpha)].$$

It can be observed from these solutions that the decaying speeds of Mittag-Leffler functions determine the speed at which $y_i(t)$, where $\text{Re}(\lambda_i) < 0$, approaches zero. As a result, it follows that the convergence speed of (7) is also determined by the decaying speeds of Mittag-Leffler functions due to the

fact shown in the proof of Theorem 1 that coordination is achieved if $y_i(t) = 0$ for all $\lambda_i \neq 0$. As a special case, for single integer-order systems (i.e., $\alpha = 1$), (7) becomes (15), and the corresponding solution is $\tilde{X}(t) = e^{-Lt} \tilde{X}(0)$. Similarly, the solution for high integer-order systems (i.e., $\alpha = 2, 3, \dots$) can also be written in the form of exponential functions. Therefore, it is worthwhile to study the difference between Mittag-Leffler functions and exponential functions to compare coordination for fractional-order dynamics and that for integer-order dynamics. As an example, we next study the decaying speeds of the Mittag-Leffler function $E_\alpha(-\lambda t^\alpha)$ and the exponential function $e^{-\lambda t}$.

Theorem 3: There exists a positive scalar T such that $E_\alpha(-\lambda t^\alpha)$ decreases faster than $e^{-\lambda t}$ for $t \in (0, T)$, where $\lambda \in \mathbb{R}^+$, and $\alpha \in \mathbb{R}^+$.

Proof: Note that both $e^{-\lambda t}$ and $E_\alpha(-\lambda t^\alpha)$ are equal to 1 when $t = 0$. Taking the derivatives of both functions gives $(d/dt)[e^{-\lambda t}]|_{t=0} = -\lambda e^{-\lambda t}|_{t=0} = -\lambda$, and $(d/dt)[E_\alpha(-\lambda t^\alpha)]|_{t=0} = -\infty$. Because $(d/dt)[e^{-\lambda t}]$ and $(d/dt)[E_\alpha(-\lambda t^\alpha)]$ are continuous with respect to t , there exists a positive scalar T such that $E_\alpha(-\lambda t^\alpha)$ decreases faster than $e^{-\lambda t}$ for $t \in (0, T)$ by using the comparison principle. ■

To illustrate, Fig. 1(a) and (b) shows, respectively, the Mittag-Leffler functions and their derivatives with different orders for $\lambda = 1$. Fig. 1(a) shows the Mittag-Leffler functions when $\alpha = 0.2i$, $i = 1, 2, 3, 4, 5$.³ A noticeable phenomenon in Fig. 1(a) is that the smaller α is, the faster the decaying speed will be when the time is close to 0. Fig. 1(b) shows the derivatives of the Mittag-Leffler functions for $\alpha = 0.2i$, $i = 1, 2, 3, 4, 5$. Note that Fig. 1(a) and (b) verified Theorem 3. Because the decaying speeds of the Mittag-Leffler functions with different fractional orders are different, as shown in Fig. 1, we are motivated to adopt a varying-order fractional-order coordination strategy to increase the convergence speed.

Remark 3: To achieve a higher convergence speed, a varying-order fractional-order coordination strategy can be adopted. The strategy can be described as follows. Let $\alpha_1 < \dots < \alpha_m < 1$, and choose α in (7) as

$$\alpha = \begin{cases} \alpha_1, & t < t_1 \\ \alpha_i, & t_{i-1} \leq t < t_i; i = 2, \dots, m \\ 1, & t \geq t_m. \end{cases}$$

Here, t_1 is chosen such that the convergence speed with order α_1 is the highest when $t < t_1$. Similarly, t_i , $i = 2, \dots, m$, is chosen such that the convergence speed with order α_i is highest for $t \in [t_{i-1}, t_i)$, and $\alpha = 1$ if $t \geq t_m$. Given the same L , the convergence speed of this varying-order fractional-order coordination strategy is higher than that of the single-integrator coordination strategy because the convergence speed of the proposed strategy is higher than that of the single-integrator coordination strategy when $t < t_m$ and equal to that of the single-integrator coordination strategy when $t \geq t_m$.

Remark 4: The convergence speed for fractional-order systems can be increased by applying a varying-order fractional-order coordination strategy. Similarly, the convergence speed

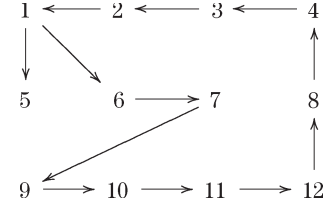


Fig. 2. Interaction graph for 12 agents. An arrow from j to i denotes that agent i can receive information from agent j .

can also be increased by separating the time interval into more pieces $[t_i, t_{i+1})$.

Remark 5: There should exist an optimal varying-order fractional-order coordination strategy to maximize the convergence speed, and the order of the corresponding fractional-order coordination strategy may be continuous with respect to t . This optimal strategy might be related to the sensitivity function of $E_\alpha(-\lambda t^\alpha)$ with respect to α , i.e., $(d/d\alpha)[E_\alpha(-\lambda t^\alpha)]$.

V. SIMULATION ILLUSTRATIONS AND DISCUSSIONS

In this section, several simulation results are presented to illustrate the fractional-order coordination algorithm proposed in Section III and the varying-order coordination strategy in Section IV. We consider a group of 12 agents with an interaction graph given by Fig. 2. Note that the interaction graph in Fig. 2 has a directed spanning tree with node 1 being the root. Although we only consider 12 agents in our simulation, similar results can be obtained for a large number of agents if the conditions in Theorem 1 are satisfied. Here, for simplicity, we have chosen $\delta_i = 0$, $i = 1, \dots, 12$, i.e., $\tilde{X}(t) = X(t)$, where $X(t) = [x_1(t), \dots, x_{12}(t)]^T$, and $\tilde{X}(t) = [\tilde{x}_1(t), \dots, \tilde{x}_{12}(t)]^T$. The corresponding (nonsymmetric) Laplacian matrix is chosen such that $a_{ij} = 1$ if $(v_j, v_i) \in \mathcal{W}$, and $a_{ij} = 0$ otherwise. It can be computed that $\mathbf{p} = [1/11, 1/11, 1/11, 1/11, 0, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11, 1/11]^T$ and the eigenvalues of L are $0, 1, 1.9595 \pm 0.2817j, 1.6549 \pm 0.7557j, 1.1423 \pm 0.9898j, 0.5846 \pm 0.9096j$, and $0.1587 \pm 0.5406j$, where j is the imaginary unit.

For $\alpha \in (0, 1]$, let the initial states be $X(0) = [6, 3, 1, -3, 4, 2, 0, -5, -2, -5, 2, 7]^T$. When the fractional order is $\alpha = 0.8$, the states using (7) are shown in Fig. 3(a). It can be seen that coordination is achieved with the final coordination equilibrium for $x_i(t)$ being 0.5455, which is equal to $\mathbf{p}^T X(0)$. When $\alpha = 1$ (i.e., the system takes in the form of single-integrator dynamics), the states using (7) are shown in Fig. 3(b). From these two figures, it can be seen that the equilibrium states for both cases are the same. In addition, it can also be observed that the convergence speed of the fractional-order case is higher than that of the single-integrator case when t is close to the origin.

For $\alpha \in (1, 2\theta/\pi)$, we let the initial states be $X(0) = [6, 3, 1, -3, 4, 2, 0, -5, -2, -5, 2, 7]^T$, and $\dot{X}(0) = [1, 2, 3, 4, 0, 0, 0, 0, 1, 1, 1, 1]^T$. It follows from the definition of θ in Theorem 1 that $\theta = 1.8563$, which implies that $\alpha \in (0, 1.182)$. Fig. 3(c) and (d) shows the states using (7) for $\alpha = 1.15$ and $\alpha = 1.5$, respectively. From Fig. 3(c), it can be observed that coordination can be achieved. From Fig. 3(d), it can be observed that

³When $\alpha = 1$, the corresponding Mittag-Leffler function becomes the exponential function.

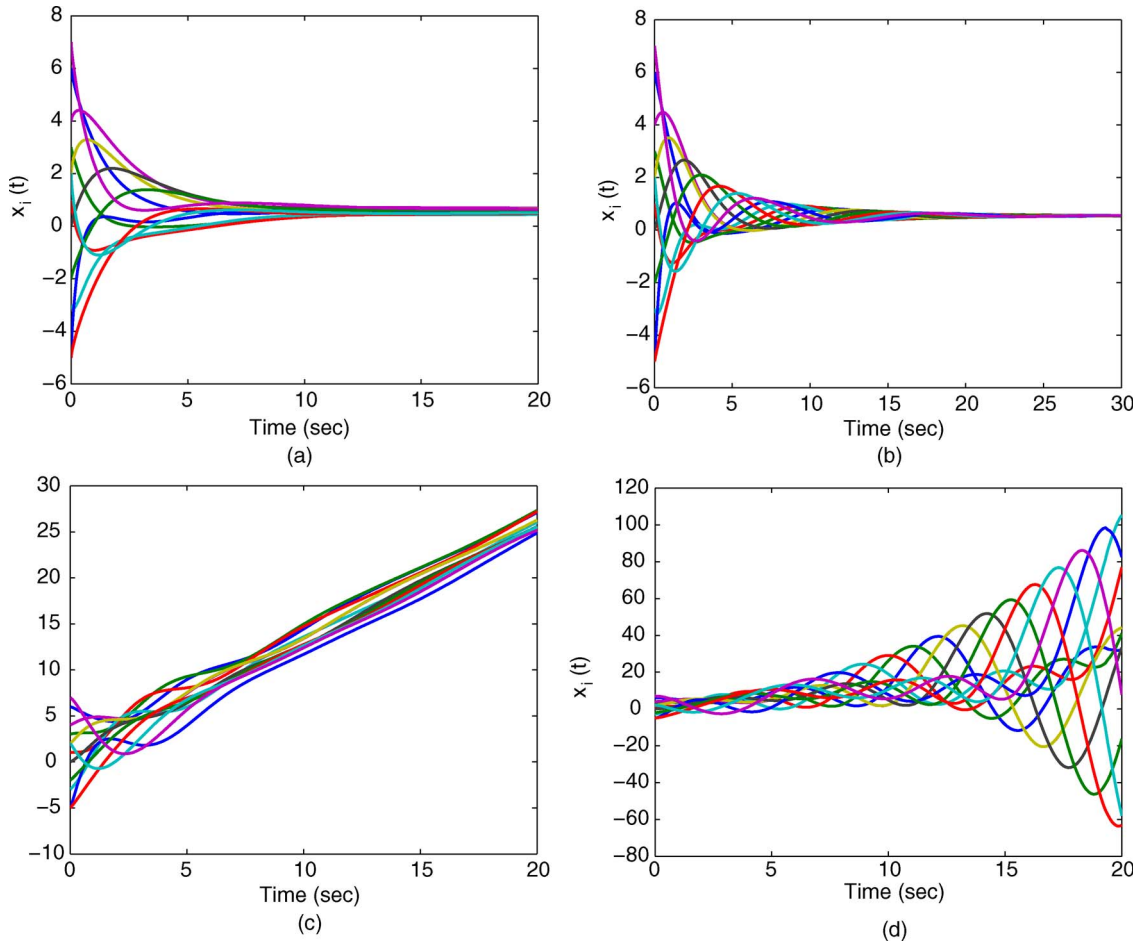


Fig. 3. Simulation results using (7) with different orders. (a) $\alpha = 0.8$. (b) $\alpha = 1$ ($|x_i(t) - x_j(t)| < 0.1$ for any $t > 22.22$ s). (c) $\alpha = 1.15$. (d) $\alpha = 1.5$.

coordination cannot be achieved. The four subfigures in Fig. 3 validate Theorem 1.

We next present the simulation results using the varying-order coordination strategy described in Remark 3 and compare the simulation results with those using the integer-order coordination strategy in Fig. 3(b). Let the initial states be $X(0) = [6, 3, 1, -3, 4, 2, 0, -5, -2, -5, 2, 7]^T$. Fig. 4 shows the states using the varying-order coordination strategy when the parameters in Remark 3 are arbitrarily chosen as $\alpha_i = 0.4 + 0.1i$ and $t_i = 0.1 + 0.04i$ for $i = 1, 2, 3, 4$. Note that $|x_i(t) - x_j(t)| < 0.1$ for all $t > 21.73$ s in Fig. 4, whereas $|x_i(t) - x_j(t)| < 0.1$ for all $t > 22.22$ s in Fig. 3(b). Therefore, we can see that the convergence speed using the varying-order coordination strategy is higher than that using the single integer-order coordination strategy. The comparison shows the effectiveness of the proposed varying-order coordination strategy. Of course, when we choose different parameters (α_i, t_i) carefully as described in Remark 3, the convergence speed can further be improved.

VI. CONCLUSION AND FUTURE WORK

In this paper, we have studied the coordination algorithms for networked fractional-order systems when the fixed interaction graph is directed. The coordination algorithms for integer-order systems can be considered a special case of those for

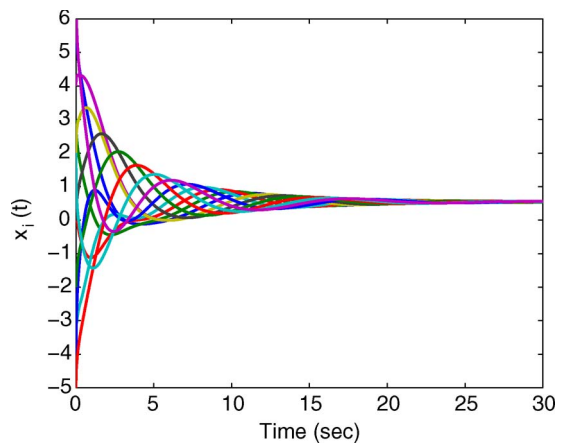


Fig. 4. Simulation result using (7) with varying orders ($|x_i(t) - x_j(t)| < 0.1$ for any $t > 21.73$ s).

fractional-order systems by letting the fractional order be integer numbers. First, we presented a general fractional-order coordination model. Then, sufficient conditions on the interaction graph and the fractional order were given to ensure coordination. In addition, we characterized the relationship between the fractional order and the number of agents to ensure coordination. Finally, a varying-order coordination strategy was proposed to improve the overall convergence speed. In

physical systems, the interaction among different agents may be dynamic due to unreliable communication, limited communication/sensing range, and/or sensing with a limited field of view. It is, therefore, meaningful to study fractional-order coordination algorithms under time-varying or switching interaction. This will be one of our future research directions.

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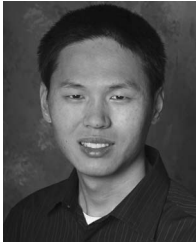
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